# Math 332 • October 23, 2009

Midterm Examination

This is a closed-book exam; neither notes nor calculators are allowed.

**1)** (24pts) For each, find **all distinct** values of $z$ in Cartesian form, and show roughly their locations as points in the complex plane:

(a) $z = (8-8i)^{2/3}$ (b) $z = 2^{i}$ (c) $\sin z = i$

**2)** (12pts) Sketch the region $1 \le |z| \le e$, Re $z > 0$, and sketch its image under the transformation $w$=Log( $z$ )

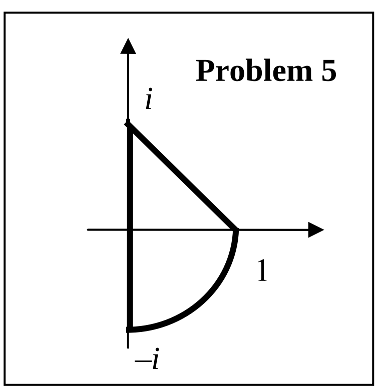
**3)** (14pts) Is function $f(z) = \operatorname{Im} z$ continuous at $z$=0? Is it differentiable at z=0? Explain your answers; use two methods to analyze differentiability: (1) the limit definition of derivative, (2) the Cauchy-Riemann equations.

**4)** (15pts) Show that the function $u(x,y) = \exp(-2y)\cos(2x)$ is harmonic; find its harmonic conjugate, $v(x,y)$, and express the function $f = u(x,y) + i\,v(x,y)$ in terms of $z$

**5)** (14pts) Use curve parametrization (instead of an anti-derivative or Cauchy theorem) to integrate $\oint_\Gamma z^2\,dz$ around the closed contour $\Gamma$ shown in the figure below. Check your answer by comparing with the exact value of this integral.

**6)** (21pts) Which of these integrals equal zero for any loop $\Gamma$ contained within the domain $|z| < 1$? Explain your answers

(a) $\displaystyle\oint_\Gamma \frac{dz}{(2z+i)^{1/2}}$ (b) $\displaystyle\oint_\Gamma \frac{dz}{\cos z}$ (c) $\displaystyle\oint_\Gamma \frac{1}{\operatorname{Log}\left(\frac{z}{2}+1\right)}\,dz$




**Alternative to problem 2**
(note the smaller number of points):

**2')** (10pts) Sketch the region $1 \le |z| \le 2$, Re $z < 0$, Im $z > 0$, and sketch its image under the transformation $w = \dfrac{i}{z^2}$